Investigation 2

Connecting Ratios and Rates

In Investigation 1, you used fraction strips as a tool to determine the fraction of each fundraising goal reached and locate points and distances on a number line. You also used ratios to compare quantities and checked to see if they were equivalent. In this Investigation you will continue to explore ratios and ways to write equivalent ratios.

The ratio statements in Investigation 1 were written as "for every" or "to" statements. Ratios can be written in many different ways.

Suppose the cost for ten students to go on a field trip is $120. You can write ratios to show how the quantities are related.

10 students *for every* $120

10 students *to every* $120

10 students : $120

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Common Core State Standards

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.A.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables . . .

6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed.

Also 6.RP.A.1, 6.RP.A.2, 6.NS.B.4

Ratio statements can also be written as "per" statements. For example, "It costs $120 per 10 students to go on the trip." An equivalent comparison statement is "the cost per student to go on a field trip is $12." Now you can say

$12 *for every* 1 student

$12 *for each* student

$12 *per* student

This particular comparison, cost per one student, is called a unit rate. A **unit rate** is a comparison in which one of the numbers being compared is 1 unit.

- If the cost of food is $250 for 50 students, what is the cost per student?

To answer this question, you find the unit rate.

2.1 Equal Shares
Introducing Unit Rates

Often we share food so that each person gets the same amount. This may mean that food is cut into smaller pieces. Think about how to share a chewy fruit worm that is already marked in equal-sized pieces.

The chewy fruit worm below shows four equal segments.

How can you share this 4-segment chewy fruit worm equally among four people?

How many segments of the worm does each person get?

OR

How can you share this 4-segment chewy fruit worm equally among three people?

How many segments of the worm does each person get?

In Questions A and B, find the fraction of a chewy fruit worm each person gets.

A 1. Show two ways that four people can share a 6-segment chewy fruit worm. In each case, how many segments does each person get?

2. Show two ways that six people can share an 8-segment chewy fruit worm. In each case, how many segments does each person get?

B 1. Show how 12 people can share an 8-segment chewy fruit worm. How many segments are there for every person?

2. Show how five people can share a 3-segment chewy fruit worm. How much is this per person?

C Jena wants to share a 6-segment chewy fruit worm. The tape diagram below shows the marks she made on the worm so she can share it equally among the members in her CMP group.

1. How many people are in her group?

2. Is there more than one possible answer to part (1)? Explain.

3. What is the number of segments per person?

4. Write a fraction to show the part of the chewy fruit worm each person gets.

D Would you rather be one of four people sharing a 6-segment chewy fruit worm or one of eight people sharing a 12-segment chewy fruit worm? Explain.

E Look back at your work on this Problem. Describe how you found or used unit rates.

A C E Homework starts on page 50.

2.2 Unequal Shares
Using Ratios and Fractions

Sometimes there are reasons to share quantities *unequally*. Suppose your older brother paid more than half the cost of a video game. You might think it is fair for him to spend more time playing the game. At a party, you might agree that your friend should take the bigger piece of chocolate cake because your friend likes chocolate more than you do.

Two sisters, Crystal and Alexa, are going to a strange birthday party. Instead of birthday cake, pairs of party guests are each served a large chewy fruit worm to share according to their ages. Since the sisters are not the same age, they do not share their fruit worm equally.

Crystal is 12 years old and Alexa is 6 years old. Their chewy fruit worm has 18 segments. According to their ages, Crystal gets 12 segments and Alexa gets 6 segments. The ratio of the girls' shares of the worm, 12 to 6, is equivalent to the ratio of their ages, 12 to 6.

- According to the rule, how would the girls share a 9-segment chewy fruit worm?

Since Crystal's age is two times Alexa's age, Crystal gets twice as many segments as Alexa. The ratio of Crystal's segments to Alexa's segments is 12 to 6 or 2 to 1.

- The ratio 2 to 1 is a unit rate. What do the numbers 2 and 1 mean for the sisters?

In this Problem you will explore situations that involve fractions and ratios.

Problem 2.2

Ⓐ Draw some chewy fruit worms with different numbers of segments that Crystal and Alexa can share without having to make new cuts.

Ⓑ 1. Jared is 10 years old. His brother Peter is 15 years old. What are some chewy fruit worms they can share without having to make new cuts?

2. For each worm you described in part (1), write a ratio comparing the number of segments Jared gets to the number of segments Peter gets.

3. Are the ratios you wrote in part (2) equivalent to each other? Explain.

4. How would you write a unit rate to compare how many segments Jared and Peter get?

Ⓒ 1. Caleb and Isaiah are brothers. They share a 14-segment chewy fruit worm according to their age. How old could they be?
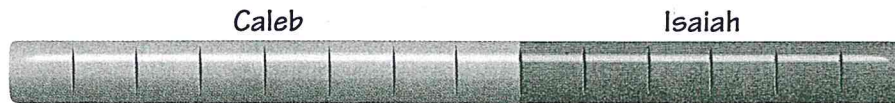
Caleb   Isaiah

2. Caleb gets 8 out of the 14 segments of the chewy fruit worm, so he gets $\frac{8}{14}$ and Isaiah gets $\frac{6}{14}$ of the worm.

a. From Question A, what fractions of the chewy fruit worm do Crystal and Alexa each get at the birthday party?

b. From Question B, what fractions of the chewy fruit worm do Jared and Peter each get at the birthday party?

c. How does the ratio of segments that Caleb and Isaiah get relate to the fractions of the chewy fruit worm that they each get?

ⒶⒸⒺ Homework starts on page 50.

2.3 Making Comparisons With Rate Tables

When comparing how to share chewy fruit worms, Crystal recorded how many segments she and her sister would get for different sizes of chewy fruit worms. Crystal thought she could use what she knew about equivalence to make a table showing the amounts.

Comparing Segments

Segments for Alexa	6	3	1	2	$\frac{1}{2}$	10
Segments for Crystal	12	6	2	4	1	20

The table shows that for every segment given to Alexa, Crystal gets two segments. This is Alexa's unit rate. The table also shows that for every $\frac{1}{2}$ segment Alexa is given, Crystal gets one segment. This is Crystal's unit rate.

Crystal sees an ad for chewy fruit worms. She decides she wants the student council to include chewy fruit worms in the fundraising sale.

Chewy Fruit Worms
LARGE SIZE
Price Reduced! 30 worms for $3

You can use the information in the advertisement to compute the price for any number of worms you want to buy. One way to figure out the price of a single item from a quantity price is use the information to build a **rate table** of equivalent ratios.

The rate table in Question A shows the price for different numbers of chewy fruit worms. The cost of 30 chewy fruit worms is $3.

Problem 2.3

A 1. Crystal wants to calculate costs quickly for many different numbers of chewy fruit worms. Copy and complete the rate table below with prices for each of the numbers of chewy fruit worms.

Chewy Fruit Worm Pricing

Number of Worms	1	5	10	15	30	90	150	180
Reduced Price	▨	▨	▨	▨	$3	▨	▨	▨

2. How much do 3 chewy fruit worms cost? 300 chewy fruit worms?

3. How many chewy fruit worms can you buy for $50? For $10?

4. What is the unit price of one chewy fruit worm? What is the unit rate?

B The student council also decides to sell popcorn to raise money. One ounce of popcorn (unpopped) kernels yields 4 cups of popcorn. One serving is a bag of popcorn that holds 2 cups of popcorn.

1. Use a rate table to find the number of ounces of popcorn kernels needed to determine the cups of popcorn.

Cups of Popcorn From Ounces of Kernels

Number of Cups of Popcorn	4	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨
Number of Ounces of Popcorn Kernels	1	2	3	4	5	6	7	8	9	10	11	12

2. How many cups of popcorn can you make from 12 ounces of popcorn kernels? From 30 ounces of popcorn kernels?

3. How many ounces of popcorn kernels are needed to make 40 cups of popcorn? To make 100 cups of popcorn?

4. How many ounces of kernels are needed to make 100 servings?

5. How many ounces of kernels are needed to make 1 cup?

C 1. How do rate tables help you answer Question A and Question B?

2. How do unit rates help you answer Question A and Question B?

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